equations as

$$\begin{aligned}
&{}_{t+\Delta t/2}H_x\left(i, j+\tfrac{1}{2}, k+\tfrac{1}{2}\right) \\
&\frac{2}{Z}\sqrt{\frac{\varepsilon_0}{\mu_0}}\Big\{{}_tE_y\left(i, j+\tfrac{1}{2}, k+1\right) \\
&\qquad -{}_tE_y\left(i, j+\tfrac{1}{2}, k\right) \\
&\qquad +{}_tE_z\left(i, j, k+\tfrac{1}{2}\right) \\
&\qquad -{}_tE_z\left(i, j+1, k+\tfrac{1}{2}\right)\Big\} \\
&+2\,{}_{t-\Delta t/2}H_x\left(i, j+\tfrac{1}{2}, k+\tfrac{1}{2}\right) \\
&-{}_{t-\Delta t/2}H_x\left(i, j+\tfrac{1}{2}, k+\tfrac{1}{2}\right).
\end{aligned}$$

This is the same as one of the finite-difference equations used by Taflove and Brodwin in [6] in their implementation of Yee's original formulation [7]. The remaining five equations can be derived in a similar way. The two-dimensional method is a simplification of the above, requiring modification of (9)–(11).

III. Comparisons Between TLM and Finite Differences

Great care has to be taken in comparing computer resources for the TLM method with the finite-difference method since much more information is available in the former. In the three-dimensional TLM method operated in the above way, there are three field quantities available at each shunt and series node. This, for example, allows the boundary description for TLM to be twice as fine as for finite differences. In two dimensions, if boundaries are described only at nodes as in finite differences, then incident pulses need only be at alternate nodes at any instant. Thus, an average of two stores for link lines, not four, is required at each node. Alternatively, if the pulses are incident simultaneously at all nodes, then boundaries can exist halfway between nodes as well as at nodes, and the boundary description is again finer than in finite differences. Also, in assessing arithmetical load, it should be recognized that implementation of (2) and (3) requires much less work than a matrix multiplication.

Comparison of the algorithm is interesting, but often there is a balance between computational efficiency and program or data complexity. A much more important difference between TLM and finite difference is that the former is a physical model using transmission lines, while the latter is a mathematical model using differencing. The advantage of TLM is that it provides the engineer with a conceptual model which can be simulated exactly on a digital computer. The comparison should include the modeling philosophy and not just the algorithm details.

Another advantage of the TLM approach is that it can lead to models and algorithms which cannot be readily expressed in terms of the field quantities because the scattering matrix is not easily factorized as in (3). Examples of this are the asymmetrical condensed node or punctual node [8], [9] and the symmetrical condensed node [10], which have the advantage of condensing all six field quantities to one point in space.

In the author's view, the TLM method and the finite difference method complement each other rather than compete with each other. Each leads to a better understanding of the other.

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Approximate Determination of the Characteristic Impedance of the Coaxial System Consisting of an Irregular Outer Conductor and a Circular Inner Conductor

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***Abstract* — An elementary formula is presented for the determination of the characteristic impedance of a coaxial transmission line consisting of a circular inner conductor and an irregular outer conductor. In this approach, the irregular outer conductor is replaced by an eccentric circular outer conductor which has the same "shield factor" as an irregular one at the extreme of a small wire, and the same formula is adapted for outer conductors of different shapes by determining values of eccentricity of the equivalent eccentric coaxial lines. The validity of the formula is confirmed by numerical results.**

I. Introduction

Considerable work has been done on the determination of the characteristic impedance of a coaxial transmission line consisting of a circular inner conductor and a noncircular outer conductor [1]–[9]. Elementary formulas for some shapes have been available for small ratios of inner and outer conductors. A formula for

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polygonal outer conductors has been derived by transitioning between the extremes of a small wire and one near contact [5]. The purpose of this paper is to derive a general and simple formula for an arbitrarily irregular outer conductor by a systematic approach. The validity of the formula is confirmed by some typical examples.

II. The Method

As we know, the interior of a unit circle in the ζ-plane can be conformally mapped into the interior of a simply connected region in the W-plane by an infinite series of the form [10]

$$W=\sum_{n=0}^{\infty}\alpha_n\zeta^{1+n} \tag{1}$$

where

$$\alpha_n=a_n+ib_n. \tag{2}$$

If $|\zeta|\ll 1$, the first term in (1) is predominant, and a circle with radius $r\ll 1$ in the W-plane will map into an approximate circle in the ζ-plane with radius

$$R=\frac{r}{|\alpha_0|}. \tag{3}$$

When the region in the W-plane has several axes of symmetry, b_n in (2) is zero. The coefficients α_n in (1) can be systematically determined by such numerical methods as successive approximations or Melentiev's method [10]. The coefficient α_0 also can be obtained by means of closed analytic functions which conformally map a unit circle in the ζ-plane into a simply connected region in the W-plane.

It is known that an arbitrarily irregular outer conductor, as shown in Fig. 1(a), can be replaced by a concentrically circular outer conductor with the effective radius

$$r_e=s(r_2/r_1)\cdot r_1 \tag{4}$$

where r_1 is the radius of the circle inscribed in the outer conductor and r_2 is the radius of the circular inner conductor. When the ratio $r_2/r_1\to 0$, r_e is given approximately from (3) by

$$r_e=|\alpha_0|r_1 \tag{5}$$

where $|\alpha_0|$ is usually referred to as a shield factor. The characteristic impedance of the line is given by

$$Z_0=59.952\ln\left(r_e/r_2\right). \tag{6}$$

Generally, it is difficult to solve for $s(r_2/r_1)$ in (4) exactly since the ratio r_2/r_1 increases for an arbitrarily irregular outer conductor. From the view that the total capacitance of a coaxial transmission line is composed of the parallel connection of the capacitance of every segment of the boundary, we have the following conclusion: The function $s(r_2/r_1)$ monotonically decreases as the ratio r_2/r_1 increases.

The above conclusion is obvious from physical considerations and has been confirmed by examples such as an eccentrical coaxial line. This leads us to propose using an eccentrically circular outer conductor to replace the irregular one shown in Fig. 1(b). The eccentrically circular outer conductor has the same shield factor as that of an irregular conductor at the extreme of a small wire and the same radius of the inscribed circle. Thus, the formula for the determination of the characteristic impedance of a coaxial transmission line consisting of a circular inner conductor and an irregular outer conductor is given by

$$Z_0=59.952\ln\left(G+\sqrt{G^2-1}\right) \tag{7}$$

(a) (b)

Fig. 1. The various radii of a cross section.

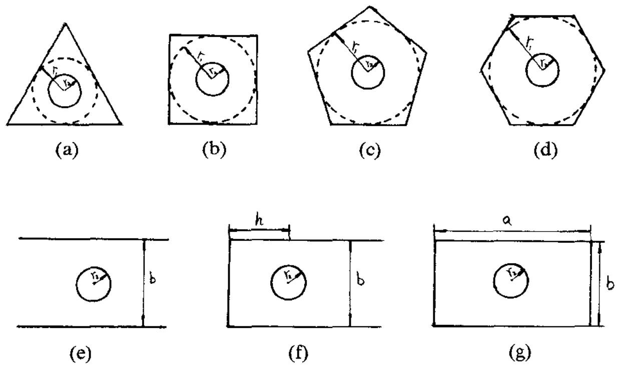

Fig. 2. Cross-sectional shapes: (a) $N=3$, $E=0.12132$; (b) $N=4$, $E=0.07874$; (c) $N=5$, $E=0.05260$; (d) $N=6$, $E=0.03772$; (e) $E=0.27313$; (f) $h/b=1/2$, $E=0.1678$; (g) $b/a=1/2$, $E=0.26404$.

where

$$G=\frac{1}{2}\left\{\frac{2r_2}{D}+\frac{D}{r_2}(1-E)[1-(1-E)/2]\right\} \tag{8a}$$

$$D=\frac{2r_1}{1-E}. \tag{8b}$$

When $2r_2/D\to 0$, the effective radius of the eccentric circular outer conductor is given simply by

$$r_e=1+E. \tag{9}$$

From the condition that (7) should equal (6) as $r_2/r_1\to 0$, we obtain

$$E=|\alpha_0|-1. \tag{10}$$

III. Examples

To show the validity of the formula obtained in the preceding section, we consider some typical examples (as shown in Fig. 2). They have been chosen because accurate approximations have been given in the literature and are available for comparison.

For each shape, α_0 in (1) is determined by numerical methods such as successive approximations, Melentiev's method, or a closed analytic function, if it exists. Then, the eccentricities E in (7) are found from (10) and are given in Fig. 2 for each shape.

The results obtained for regular polygons are compared with the earlier published results in Table I for different ratios r_2/r_1. Table II shows the numerical results obtained for shapes (e)–(g) shown in Fig. 2. It is seen that the values given here are in good agreement with those reported in the literature.

IV. Conclusion

A method was presented for the approximate equivalence of an irregular outer conductor. A general formula was given for the determination of the characteristic impedance of a coaxial line consisting of an irregular outer conductor and a circular inner conductor. It was shown by numerical examples that the formula yields accurate results in most cases for different ratios of r_2/r_1.

TABLE I
THE CHARACTERISTIC IMPEDANCE OF THE POLYGONAL LINE WITH CIRCULAR INNER CONDUCTOR

	shape (a) N=3			shape (b) N=4			
r_2/r_1	present work	Seshadri and Rajaian[6]	Lin[8]	present work	Seshadri and Rajaian[6]	Lin[8]	Riblet[3]
0.05	186.46	187.32	185.16	184.14	184.42	183.77	
0.1	144.90	145.70	144.36	142.59	142.50	142.21	
0.2	103.32	104.08	102.05	101.02	101.18	100.66	
0.3	78.97	79.74	77.74	76.69	76.84	76.35	
0.4	61.66	62.45	60.49	59.42	59.56	59.10	
0.5	48.20	49.03	47.12	46.00	46.16	45.73	46.09
0.6	37.13	38.01	36.19	35.00	35.20	34.80	35.15
0.7	27.67	26.57*	26.94	25.66	25.89	25.55	25.85
0.8	19.30	20.14	18.94	17.46	17.71	17.55	17.68
0.9	11.47	12.06	11.88	9.97	10.15	10.49	10.13
0.95	7.34		8.64	6.20		7.25	6.25
0.99	3.02		6.17				
0.998	1.24		5.68	1.07		4.29	1.00
	shape (c) N=5			shape (d) N=6			
r_2/r_1	present work	Seshagiri [1]	Lin[8]	present work	Seshagiri [1]	Lin[8]	
0.05	182.67	182.58	182.94	181.82	181.64	182.38	
0.1	141.12	140.92	141.38	140.26	140.01	140.82	
0.2	99.52	99.26	99.83	98.71	98.38	99.27	
0.3	75.26	74.91	75.52	74.39	74.04	74.96	
0.4	57.98	57.65	58.27	57.14	56.79	57.71	
0.5	44.58	44.30	44.89	43.75	43.43	44.34	
0.6	33.62	33.42	33.97	32.80	32.58	33.41	
0.7	24.32	24.24	24.72	23.53	23.34	24.16	
0.8	16.22	16.26	16.72	15.47	15.37	16.16	
0.9	8.93	9.06	9.66	8.27	8.25	9.10	
0.94	6.09	6.23	7.05	5.53	5.52	6.49	
0.99	2.04		3.94	1.76		3.39	

* It should read 28.57

TABLE II
THE CHARACTERISTIC IMPEDANCE OF THE SHAPES (e)–(g)

	shape (e) parallel plates			shape (f) trough (b=2h)		shape (g) rectangle (a=2b'		
2r/b	present work	Wheeler [1]	Lin [1]	present work	Chisholm [1]	present work	Lin and Chuang[1]	Pan [9]
0.05	194.07			188.90	188.89	193.64	193.64	193.64
0.1	152.49	152.54	152.53	147.34	147.33	152.07	152.08	152.13
0.2	110.85	110.98	110.97	105.74	105.74	110.43	110.52	110.56
0.3	86.39	86.63	86.64	81.37	81.36	85.98	86.19	86.15
0.4	68.92	69.29	69.34	64.01	64.01	68.51	68.89	68.72
0.5	55.21	55.72	55.84	50.48	50.46	54.82	55.39	55.07
0.6	43.80	44.42	44.65	39.32	39.28	43.44	44.21	43.72
0.7	33.86	34.52	34.91	29.72	29.67	33.53	34.48	33.86
0.8	24.75	25.35	25.93	21.13	21.12	24.46	25.52	24.82
0.9	15.68	16.03	16.78	12.91	13.24	15.46	16.44	15.73
0.94				9.40	10.19			
0.95	10.49	10.58	11.37			10.32	11.11	10.46
0.99	4.49	4.17	4.92	3.53		4.41	4.80	4.39

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A Simplified Large-Signal Simulation of a Lumped Element TEO Based on a Phase Plane Technique

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***Abstract* —The transferred electron device (TED) lumped circuit interaction is modeled by a phase plane technique. The results from this large-signal simulation are compared to those from a time-domain simulation based on the electron transport equations and are shown to be in good agreement. Results from the simulations are used as the design specifications for a *J*-band MIC TEO with excellent results indicating the potential of this CAD technique.**

I. INTRODUCTION

The microwave performance characteristics and frequency of operation of a transferred electron device (TED) are dependent not only on the physical parameters of the TED but also on the external circuit in which the device is embedded. Several studies [1]–[3] have attempted to simplify the characterization of this highly nonlinear device–circuit interaction. Simple analysis yields some understanding of the TED behavior but has been of little direct use to the design engineer who has relied mainly on indirect measurements [4]–[6] of the admittance of a particular diode at a specific frequency of operation, to optimize performance. Only in the case of LSA oscillator design has a simplified analysis been of any significance. In the LSA mode [7], associated with oversize n-GaAs samples, space-charge accumulation is negligable and hence an assumption that a uniform electric-field profile exists across the device allows the voltage V_D and average electron current $\bar{I}_e$ to be linked by a piecewise-linear approximation to the electron drift velocity-electric field (v-E) characteristic of n-GaAs. This assumption is, however, invalid for short, commercial TED's in which space-charge accumulation is significant.

Large-signal time-domain simulations which are based on numerical solutions of the electron transport equations, have the advantage of assuming only the physical properties of GaAs and the geometry and doping of the TED. External circuit elements and a bias can be incorporated into these numerical schemes which are run until steady-state oscillations occur when a Fourier analysis of the voltage and current waveforms yield the frequency of operation, dynamic device admittance, RF power, and dc to RF conversion efficiency. Several large-signal time domain simulations have been developed [8]–[10] to determine device performance and operation but have not been compared to experimental data. Lakshminarayana and Partain [6] have, however, produced reasonable agreement between the results of their large-signal time-domain simulation [11] and dynamic device admittance data measured from diodes embedded in Sharpless Flange mounts. These large-signal time-domain simulations have the flexibility to allow both device and circuit parameters to be readily adjusted to optimize oscillator performance. However, the simulations are not only difficult to establish but require significant CPU time for each run and also many runs are required since the TED's dynamic admittance, and hence the frequency of operation cannot be preset.

This paper describes the simplified method of solving the TED lumped circuit interaction using a phase plane method [12]. These

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